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Evaluation of concepts of fin efficiency using Monte Carlo Simulation method

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Abstract

Monte Carlo Simulation (MCS) method is used to evaluate the three classical formulations of fin efficiency and it has been identified that the conventional formulations either under estimate or over estimate the efficiency of typical fins. It is also shown that Gardner's model of fin efficiency holds good for a fixed point and for all values above this point, the model seems to under predict fin efficiency. It is also seen that the underlying assumptions in the model form the reason behind the same and the effect of violating each of the assumption is shown for all possible ranges of input values. A modified formulation for fin efficiency, eliminating the underlying assumptions, is also proposed.

Keywords: Monte Carlo Simulation; Murray Gardner assumptions; Fin efficiency; Performance; Cooling and dehumidifying coils

1. Introduction

Gardner in 1945 [1] gave a comprehensive form to the concept of fin efficiency covering all forms of fin geometry. However, the importance of considering moist air properties along with surface temperature was identified by McQuiston [2] in his first attempt to model wet fin efficiency. However, both these important models are based on the classical Murray Gardner assumptions, which tend to significantly deviate the obtained results from the real time solutions. The third model under consideration is proposed by Huang and Shah in 1992 [3] which considers both dry and wet fin efficiencies integrated in the same model. The testing range of input values for any such models is vast and an easier and quicker way of attempting a validation is needed. This paper uses Monte Carlo Simulation (MCS) method to evaluate three such formulations and attempts to also extend the usage of the tool to enhance the prediction of the models through stepwise violation of the underlying assumptions.

2. Methodology

A total of eight simulations were carried out using Gardner's formulation. The first simulation involves the base Gardner's model across the entire range of input variables defined through earlier literature ranging from thin fins in practice with a thickness of 0.00015 m to thick fin walls of 0.1 m thickness. The second simulation attempts to study the impact of corrected fin height on fin efficiency as proposed by Harper and Brown [4]. The corrected fin height then becomes

$$b_c = b + \frac{\delta}{2} = b + \frac{h_{\mathrm{av}}}{km^2} \tag{1}$$

The third simulation considers the temperature dependence of the fin's thermal conductivity, which could be written as

$$k = k_{\mathrm{s}}[1 + \beta(T - T_{\mathrm{s}})] \tag{2}$$

where β denotes the ratio of tip radius to base radius [5]. Typically, for a longitudinal rectangular fin, the value of

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β is 1. The fourth and fifth simulation uses a varying heat transfer coefficient using the power law proposed by Han and Lefkowitz [6],

$$h_{\text{new}} = (\gamma + 1)h_{\text{av}}\left(\frac{x}{b}\right)^{\gamma} \tag{3}$$

where γ is a number, which could be 1 or vary between 1 and 2. When γ is 1, the variation is linear and when it varies between 1 and 2, it is assumed to be parabolic. The sixth simulation uses the combined effect of temperature dependent thermal conductivity and linearly varying heat transfer coefficient as the variation. The seventh uses a parabolically varying heat transfer coefficient and the eighth simulation explores the combined effect of all the three parameters. Attempt has been made to explore McQuiston equation over an entire range temperature and RH conditions. Variation in terms of corrected fin height as well as the variable heat transfer coefficient has been attempted as before. Huang and Shah's model for fin efficiency was modified by using a variable thermal conductivity as well as a varying aspect ratio and the impact of the same on the fin efficiency was studied across different fin thicknesses under all possible inlet conditions. No heat generation is assumed in this model and the model was optimized considering a new parameter $\kappa = [\beta_1(T - T_s)]$ and the new fin efficiency obtained was expressed as

$$\eta_{\text{HS}} = \frac{\tanh(mb)}{mb} + \frac{\kappa\tanh^3(mb)}{3mb} \tag{4}$$

The model proposed by Huang and Shah uses an equation similar to the one proposed by Han and Lefkowitz to obtain the fin efficiency values with variable heat transfer coefficient. However, for the purpose of this paper, the original equation as proposed by Han and Lefkowitz is considered.

3. Results and discussion

The three models explained above have underlying assumptions which result in values distinct from real time solutions. Violation of these assumptions would make the model predict fin efficiency values closer to the actual values and this would help in the design of coils in an optimized manner. The results presented in Table 1 cover the entire range of inlet conditions for the Gardner's model, practically across most applications in heat exchangers for a fin thickness of 7 mm. It is clearly seen from the results that correcting the fin height would cause a reduction of fin efficiency from the conventional method in the range of 0.2–6% depending on varied inlet conditions and thicknesses. The third simulation shows that variable thermal conductivity has an impact in the range of 4–16%, with pronounced deviations at lower fin thicknesses. It was also noted that the variation did not have any effect on the model at a particular point (Fig. 1) which seems to be shifting towards the higher values of fin efficiency with increasing thicknesses. In a fin of 7 mm thickness, this point occurs at an approximate η value of 0.4 while it shifts gradually to a η value of 0.85 for a fin 100 mm thick. This point seems more like a point of intersection (Point of Contra-effect) of the Base Gardner's model and the modified model. Below this point, it is evidently seen that Gardner's model underestimates the values of fin efficiency and above this point, it overestimates the values of η. Considering this point as an optimum fin efficiency value, it is clear that for all desirable efficiencies in design calculation, the conventional model as used by Gardner is an over estimate of the actual heat transfer, hence leading to a poor design. Hence, application of such violations would lead to an optimization of the fin design. The results obtained from MCS studies using a variable heat transfer coefficient show results similar to earlier works done by Mokheimer [7] in

Table 1
Percentage of error between the Gardner's value of fin efficiency (η_G) and the values obtained from MCS in the eight simulations (η_{MCS}) for various fin thicknesses

Fin thickness	Range of values	Cases 1&2	Cases 1&3	Cases 1&4	Cases 1&5	Cases 1&6	Cases 1&7	Cases 1&8
Percentage of error between η_G and η_{MCS}								
0.007	Min	−0.6845	−12.4137	−11.3992	−35.3961	−44.5479	−32.8446	−38.8537
	Max	5.9730	15.6105	17.4385	13.1852	16.4134	19.7254	35.4711
0.009	Min	−1.4911	−9.5876	−11.8876	−27.3762	−37.6172	−28.4616	−31.2920
	Max	3.2327	16.8597	16.7046	12.5972	16.9141	24.1590	28.0534
0.012	Min	−1.0001	−6.7650	−7.3369	−20.5256	−28.2637	−18.7774	−24.4877
	Max	3.0899	14.9700	16.2174	14.1528	17.3589	23.6191	31.8235
0.0135	Min	−0.2213	−4.6175	−4.8320	−20.1990	−26.2962	−19.7208	−22.3144
	Max	2.7344	14.5783	16.7249	10.6486	16.9207	26.7994	31.756
0.025	Min	−0.2982	−4.5767	−3.4060	−11.1742	−14.3657	−11.7972	−10.1269
	Max	2.7146	13.2843	15.0420	8.5572	16.4015	16.4556	24.2503
0.050	Min	0.8966	−2.5615	−0.5494	−5.8823	−8.2873	−6.1895	−2.7682
	Max	4.3046	11.5719	13.7078	9.5998	10.1042	20.8870	37.6424
0.075	Min	0.9460	−1.6281	0.2859	−3.6790	−5.2660	−3.5680	−0.6672
	Max	3.4171	11.1118	15.0369	7.0784	8.5731	17.4073	42.1173
0.1	Min	1.1142	−1.3306	0.6265	−3.2869	−4.2349	−3.2891	−0.0603
	Max	5.2452	8.5458	10.0981	5.6419	8.7824	13.9198	46.8520

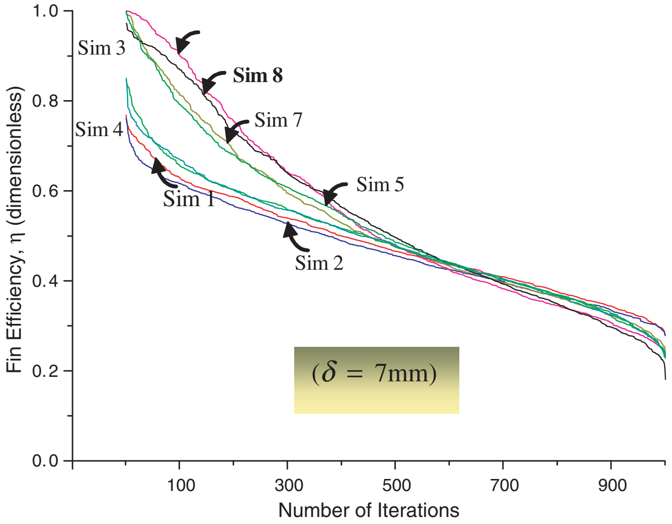

Fig. 1. Variations in calculated fin efficiencies under different simulations (Gardner's simulation).

which a comparison with Gardner's results resulted up to a 40% error in annular fins. A linear variation assumed in simulation five showed an error range between 3% and 35% (Table 1) while a parabolic distribution showed an error range of 3–45% (Table 1) across the different fin thickness. Practically, thin fins are used for cooling and dehumidifying coils and the effects of simulation three, four and five appear to be significantly high at smaller thicknesses. This would suggest that for practical cooling and dehumidifying designs, considerations of these violations would have significant optimizing opportunities. The above simulations also exhibit similar points of contra-effect and also suggest typically same optimum values of fin efficiency for the given thickness.

Dry bulb temperature (DBT) and relative humidity (RH) play an important role as input values in the McQuiston's model for fin efficiency. The results for the three different variations namely corrected fin height and variable heat transfer coefficient (linear and parabolic) was tested across the entire range of RH values for each DBT value from 22 to 28 °C. Table 2 shows the effect of corrected fin height and Table 3 shows the effect of variable heat transfer coefficient. It is seen that corrected fin height has significant effects only at lower and higher temperatures, whereas in the middle range of inlet temperatures, the effect seems to be minimal. Linear variation of heat transfer coefficient causes a maximum deviation of up to 32% whereas, a parabolic variation causes a maximum deviation of up to 42% (Table 3). However, the average values between the linear and parabolic differ by only 5%. Hence, it could be argued that a linear variation is more representative as a difference of 5% in the range of 0–1 is highly insignificant.

Aspect ratio and Biot number are the two important parameters based on which Huang and Shah's model is formulated. Table 4 shows that the change in thermal conductivity results in deviations at high values of η with a point of contra-effect in the mid range and a similar trend of over prediction without the variation beyond the optimal value for typical cooling coils. It is seen that variable heat transfer coefficient has a direct relation with the aspect ratio of the fin and the deviations are more pronounced at higher aspect ratios than the lower ones (Table 5). Hence, it is also clear that the original model proposed works well typically

Table 2
Percentage of error based on McQuiston's model from the MCS obtained values with a corrected fin height at different temperatures

	22 °C	23 °C	24 °C	25 °C	26 °C	27 °C	28 °C
Error percentage by corrected fin height							
Min	−2.3252	−3.0615	−2.2005	−0.9664	−0.4540	−1.6102	−0.2788
Max	1.3336	0.2521	0.6414	0.9321	1.0247	0.7525	4.6486

Table 3
Percentage of error based on McQuiston's model from the MCS obtained values with a variable heat transfer coefficient at different γ values

	$\gamma = 1$	$\gamma = 1.1$	$\gamma = 1.2$	$\gamma = 1.3$	$\gamma = 1.4$	$\gamma = 1.5$	$\gamma = 1.6$	$\gamma = 1.7$	$\gamma = 1.8$	$\gamma = 1.9$	$\gamma = 2$
Error percentage by variable heat transfer coefficient											
Min	3.2772	3.0196	2.7377	2.4352	2.1156	1.7817	1.4363	1.0819	0.7207	0.3545	0.0147
Max	32.9525	34.9045	36.5227	37.8519	38.9354	39.8127	40.5189	41.0848	41.5363	41.8952	42.1798

Table 4
Percentage of error based on Huang and Shah's model from the MCS obtained values based on variable parameter κ (thermal conductivity) for different fin thicknesses

	0.007	0.009	0.012	0.0135	0.025	0.050	0.075	0.1
Error percentage by variable thermal conductivity parameter								
Min	−11.9059	−19.5756	−7.7857	−14.7515	−11.8476	−7.6007	−6.4779	−4.0031
Max	12.5765	13.3801	15.5114	13.6788	15.5426	16.1123	13.8072	13.3992

Table 5
Percentage of error based on Huang and Shah's model from the MCS obtained values with a variable heat transfer coefficient under different aspect ratios

Error analysis			
Aspect ratio	Range of values	Cases 1&2	Cases 1&3
		Constant *h*-value and a linearly varying *h*-value	Constant *h*-value and a parabolically varying *h*-value
$K = 1$	Min	−0.2743	−0.4105
	Max	6.1774	10.7335
$K = 5$	Min	−3.6199	−6.2393
	Max	28.6500	52.3637
$K = 10$	Min	−10.8249	−16.6967
	Max	35.6633	56.7018
$K = 50$	Min	−46.1377	−58.9512
	Max	39.7425	49.3460
$K = 100$	Min	−29.8268	−56.1650
	Max	41.5944	49.2163

for fins with an aspect ratio of 1. Thus, based on the above discussion, it is evident that the current models need refinement in order to optimize the values of fin efficiency computed. It is also evident that MCS could be a useful tool to simulate such optimizations with known relations.

The next step in this study is to extend the application of MCS to provide a modified equation for the prediction of fin efficiency for any new cooling coil with due consideration to the three underlying assumptions and their variations as discussed above. The proposed numerical model starts from the McQuiston's equation and incorporates the three variations to the conventional model namely corrected fin height, linearly varying heat transfer coefficient and a temperature dependent thermal conductivity in the model, accounting for both dry and wet fin efficiency.

$$\eta_{\text{new}} = \frac{\tanh\left[M\left(l + \frac{h}{km^2}\right)\right]}{\left[M\left(l + \frac{h}{km^2}\right)\right]} \tag{5}$$

where

$$M = \left[\left(\frac{(\gamma+1)h\left(\frac{x}{b}\right)^{\gamma}P}{\frac{k}{[1+\beta(T-T_a)]}A}\right)\left(1 + \frac{Ci_{fg}}{C_p}\right)\right]^{1/2} \tag{6}$$

4. Conclusion

The complexity of the phenomenon of heat and mass transfer in a typical cooling and dehumidifying coil requires complicated numerical models to explain the actual phenomenon. It is difficult to empirically establish the performance of coil across all the configurations of the coil and inlet conditions. The potential use of Monte Carlo Simulation method to evaluate formulations explaining the physical phenomenon of heat and mass transfer in a typical cooling and dehumidifying coil has been illustrated in this paper. Three well established models for calculating fin efficiency have been evaluated and it has been identified that the ideal Murray Gardner assumptions, based on which these models have been formulated, needs to be violated. Violation of two of these assumptions results up to a cumulative deviation of 40% and when such a large error occurs in the process of coil design, it leads to poor performance of the system. In the context of obtaining an optimized design, it has been shown that such idealized assumptions result in under prediction of the fin efficiency values, which again would result in a poorly optimized design. A simplified numerical model considering the different violations has been formulated and it has been shown that this model results in a cumulative deviation up to 40%. Apart from evaluating existing models, the approach of MCS can be used to predict the performance of newly developed coils with a few empirical results to validate the results so obtained. Thus, the application of this tool for explaining complex physical phenomenon can be considered as an easy and efficient solution in comparison with the conventional empirical method.

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